Transformadas de Laplace básicas (1)

Señal	Transformada	ROC
$x(t) = \frac{1}{2\pi j}\int_{\sigma-j\infty}^{\sigma+j\infty} X(s)e^{st}ds$	$X(s) = \int_{-\infty}^{\infty} x(t)e^{-st}dt$	
u(t)	$\frac{1}{s}$	Re{s}>0
t u(t)	$\frac{1}{s^2}$	Re{s}>0
$\delta(t-\tau), \tau \geq 0$	e^{-st}	$\forall\, s$
$e^{-at}u(t)$	$\frac{1}{s+a}$	Re{s}>-a
$te^{-at}u(t)$	$\frac{1}{(s+a)^2}$	Re{s}>-a

Transformadas de Laplace básicas (2)

Señal	Transformada	ROC
$x(t) = \frac{1}{2\pi}\int_{\sigma-j\infty}^{\sigma+j\infty} X(s)e^{st}ds$	$X(s) = \int_{-\infty}^{\infty} x(t)e^{-st}dt$	
$[\cos(\omega_1 t)]u(t)$	$\frac{s}{s^2+\omega_1^2}$	Re{s}>0
$[sen(\omega_1 t)]u(t)$	$\frac{\omega_1}{s^2+\omega_1^2}$	Re{s}>0
$[e^{-at}\cos(\omega_1 t)]u(t)$	$\frac{s+a}{(s+a)^2+\omega_1^2}$	Re{s}>-a
$[e^{-at}sen(\omega_1 t)]u(t)$	$\frac{\omega_1}{(s+a)^2+\omega_1^2}$	Re{s}>-a
$t^n u(t)$	$\frac{n!}{s^{n+1}}$	Re{s}>0

Transformadas de Laplace bilaterales,
para señales distintas de cero para $t \leq 0$

Señal	Transformada	ROC
-u(-t)	$\frac{1}{s}$	Re{s}<0
-t u(-t)	$\frac{1}{s^2}$	Re{s}<0
$\delta(t-\tau),\ \tau \leq 0$	e^{-st}	$\forall\ s$
$-e^{-at}u(-t)$	$\frac{1}{s+a}$	Re{s}<-a
$-te^{-at}u(-t)$	$\frac{1}{(s+a)^2}$	Re{s}<-a

Propiedades de la transformada de Laplace (1)

Señal	Transformada unilateral	Transformada bilateral	ROC
x(t)	X(s)	X(S)	$s \in R_x$
y(t)	Y(s)	Y(S)	$s \in R_y$
ax(t)+by(t)	aX(s)+bY(s)	aX(s)+bY(s)	al menos $R_x \cap R_y$
x(t-τ)	$e^{-s\tau}X(s)$ si $x(t-\tau)u(t) = x(t-\tau)u(t-\tau)$	$e^{-s\tau}X(s)$	R_x
$e^{-s_0 t}x(t)$	$X(s-s_0)$	$X(s-s_0)$	$R_x + \text{Re}\{s_0\}$
x(at)	$\frac{1}{a}X\left(\frac{s}{a}\right),\ a>0$	$\frac{1}{\lvert a\rvert}X\left(\frac{s}{a}\right)$	$\frac{R_x}{\lvert a\rvert}$

Propiedades de la transformada de Laplace (2)

Señal	Transformada unilateral	Transformada bilateral	ROC
x(t)	X(s)	X(S)	$s \in R_x$
y(t)	Y(s)	Y(S)	$s \in R_y$
x(t)*y(t)	X(s)Y(s)	X(s)Y(s)	al menos $R_x \cap R_y$
-t x(t)	$\frac{d}{ds}X(s)$	$\frac{d}{ds}X(s)$	R_x
$\frac{d}{dt}x(t)$	$sX(s)-x(0^+)$	$sX(s)$	al menos R_x
$\int_{-\infty}^{t} x(\tau)d\tau$	$\frac{1}{s}\int_{-\infty}^{0^-} x(\tau)d\tau + \frac{X(s)}{s}$	$\frac{X(s)}{s}$	al menos $R_x \cap \{\mathrm{Re}\{s\}>0\}$

Propiedades de la transformada de Laplace (3)

Teorema del valor inicial $\lim_{s\to\infty} sX(s) = x(0^+)$

Teorema del valor final $\lim_{s\to 0} sX(s) = x(\infty)$ solo si Re(polos)<0

Propiedad de diferenciación unilateral-Forma general

$$\frac{d^n}{dt^n}x(t) \xleftrightarrow{L_u} s^n X(s) - \frac{d^{n-1}}{dt^{n-1}}x(t)\Big|_{t=0^+} - \ldots - s^{n-2}\frac{d}{dt}x(t)\Big|_{t=0^+} - s^{n-1}x(0^+)$$