Problema 1.44

Considere la señal sinusoidal : $x(t) = A\cos(\omega t + \phi)$

Determine la potencia media de x(t)

$$x(t+T) = A\cos[\omega(t+T)+\phi] = A\cos(\omega t + \phi + T\omega) \Rightarrow T\omega = 2\pi \Rightarrow T = \frac{2\pi}{\omega}$$

$$Potencia \quad P = \frac{1}{T}\int_{-T/T}^{T/2} x^2(t)dt = \frac{\omega}{2\pi}\int_{-\pi/\omega}^{\pi/\omega} A^2\cos^2(\omega t+\phi)dt$$

$$P = \frac{A^2\omega}{2\pi}\int_{-\pi/\omega}^{\pi/\omega}\frac{1}{2}\left[1-\cos 2(\omega t+\phi)\right]dt = \frac{A^2\omega}{4\pi}\int_{-\pi/\omega}^{\pi/\omega} dt - \frac{A^2\omega}{4\pi}\frac{1}{2\omega} sen 2(\omega t+\phi)\Big|_{-\frac{\pi}{\omega}}^{\frac{\pi}{\omega}}$$

$$P = \frac{A^2}{2} - \frac{A^2\omega}{8\pi}\left[sen(2\pi+\phi) - sen(-2\pi+\phi)\right] = \frac{A^2}{2}$$

$$P = \frac{A^2}{2}$$

Problema 1.64

1.64 The systems that follow have input $x(t)$ or $x[n]$ and output $y(t)$ or $y[n]$. For each system, determine whether it is (i) memoryless, (ii) stable, (iii) causal, (iv) linear, and (v) time invariant.

(a) $y(t) = \cos(x(t))$

(b) $y[n] = 2x[n]u[n]$

(c) $y[n] = \log_{10}(|x[n]|)$

(d) $y(t) = \int_{-\infty}^{t/2} x(\tau)\,d\tau$

(e) $y[n] = \sum_{k=-\infty}^{n} x[k+2]$

(f) $y(t) = \frac{d}{dt}x(t)$

(g) $y[n] = \cos(2\pi x[n+1]) + x[n]$

(h) $y(t) = \frac{d}{dt}\{e^{-t}x(t)\}$

(i) $y(t) = x(2-t)$

(j) $y[n] = x[n]\sum_{k=-\infty}^{\infty}\delta[n-2k]$

(k) $y(t) = x(t/2)$

(l) $y[n] = 2x[2^n]$

1.64	Memoryless	Stable	Causal	Linear	Time-invariant
(a)	✓	✓	✓	x	✓
(b)	✓	✓	✓	✓	✓
(c)	✓	✓	✓	x	✓
(d)	x	✓	✓	✓	✓
(e)	x	✓	x	✓	✓
(f)	x	✓	✓	✓	✓
(g)	✓	✓	x	x	✓
(h)	x	✓	✓	✓	✓
(i)	x	✓	x	✓	✓
(j)	✓	✓	✓	✓	✓
(k)	✓	✓	✓	✓	✓
(l)	✓	✓	✓	x	✓

Problema 1.65

1.65 The output of a discrete-time system is related to its input $x[n]$ as follows:

$$y[n] = a_0x[n] + a_1x[n-1] + a_2x[n-2] + a_3x[n-3].$$

Let the operator S^k denote a system that shifts the input $x[n]$ by k time units to produce $x[n-k]$. Formulate the operator H for the system relating $y[n]$ to $x[n]$. Then develop a block diagram representation for H, using (a) cascade implementation and (b) parallel implementation.

Problema 1.65

$$S^k\{x(n)\} = x(n-k)$$

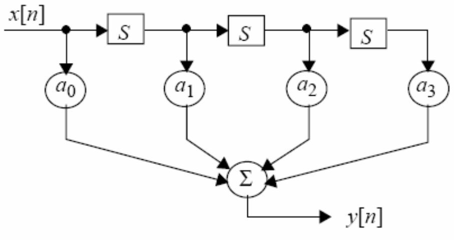
We may then rewrite Eq. (1) in the equivalent form

$$y[n] = a_0x[n] + a_1S^1\{x[n]\} + a_2S^2\{x[n]\} + a_3S^3\{x[n]\}$$
$$= (a_0 + a_1S^1 + a_2S^2 + a_3S^3)\{x[n]\}$$
$$= H\{x[n]\}$$

where

$$H = a_0 + a_1S^1 + a_2S^2 + a_3S^3$$

(a) Cascade implementation of operator H:



Problema 1.76

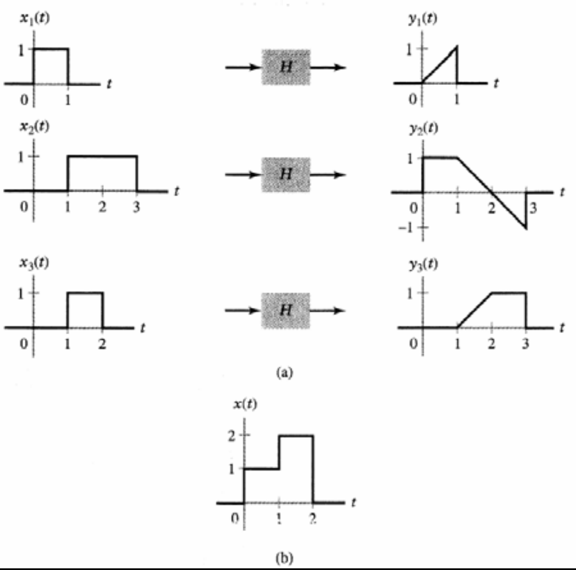
1.76 A linear system H has the input–output pairs depicted in Fig. P1.76(a). Answer the following questions, and explain your answers:

(a) Could this system be causal?

(b) Could this system be time invariant?

(c) Could this system be memoryless?

(d) What is the output for the input depicted in Fig. P1.76(b)?



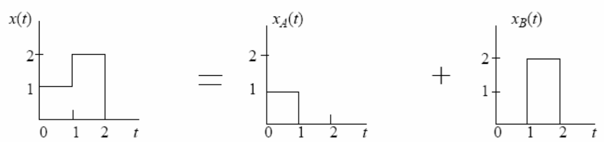
Problema 1.76

1.76 H_1 is representative of an integrator, and therefore has memory. It is causal because the output does not appear before the input. It is time-invariant.

H_2 is noncausal because the output appears at $t = 0$, one time unit before the delayed input at $t = +1$. It has memory because of the integrating action performed on the input. But, how do we explain the constant level of +1 at the front end of the output, extending from $t = 0$ to $t = +1$? Since the system is noncausal, and therefore operating in a non real-time fashion, this constant level of duration 1 time unit may be inserted into the output by artificial means. On this basis, H_2 may be viewed as time-varying.

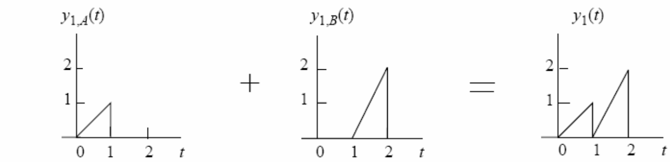
H_3 is causal because the output does not appear before the input. It has memory because of the integrating action performed on the input from $t = 1$ to $t = 2$. The constant level appearing at the back end of the output, from $t = 2$ to $t = 3$, may be explained by the presence of a strong device connected in parallel with the integrator. On this basis, H_3 is time-invariant.

Consider next the input $x(t)$ depicted in Fig. P1.76b. This input may be decomposed into the sum of two rectangular pulses, as shown here:

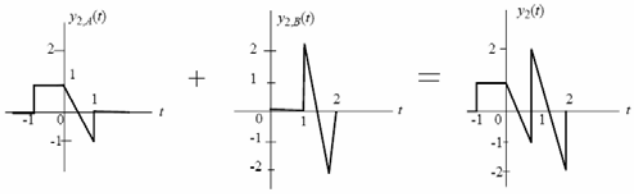


Problema 1.76

Response of H_1 to $x(t)$:

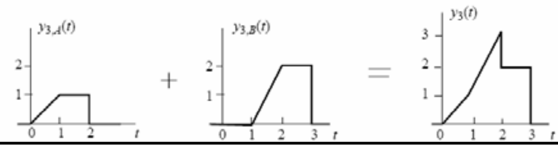


Response of H_2 to $x(t)$:



The rectangular pulse of unit amplitude and unit duration at the front end of $y_2(t)$ is inserted in an off-line manner by artificial means

Response of H_3 to $x(t)$:



4.25. Consider sampling the signal $x(t) = \frac{1}{\pi t}\sin(2\pi t)$.

(a) Sketch the FT of the sampled signal for the following sampling intervals:

(i) $T_s = \frac{1}{8}$

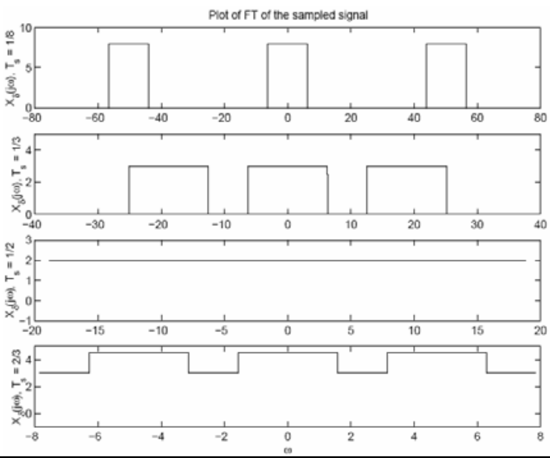
(ii) $T_s = \frac{1}{3}$

(iii) $T_s = \frac{1}{2}$

(iv) $T_s = \frac{2}{3}$

$$x(t) = \frac{1}{\pi t} sen(Wt) \xrightarrow{FT} X(j\omega) = \begin{cases} 1, & |\omega| \le W \\ 0, & en\ otro\ caso \end{cases}$$

In part (iv), aliasing occurs. The signals overlap and add, which can be seen in the following figure.



Transformadas de z básicas (1)

Señal	Transformada	ROC
$x[n]=\frac{1}{2\pi j}\oint X(z)z^{n-1}dz$	$X[z]=\sum_{n=-\infty}^{\infty}x[n]z^{-n}$	
$\delta[n]$	1	Toda z
$u[n]$	$\frac{1}{1-z^{-1}}$	$\lvert z\rvert>1$
$\alpha^n u[n]$	$\frac{1}{1-\alpha z^{-1}}$	$\lvert z\rvert>\lvert\alpha\rvert$
$n\alpha^n u[n]$	$\frac{\alpha z^{-1}}{(1-\alpha z^{-1})^2}$	$\lvert z\rvert>\lvert\alpha\rvert$
$[\cos(\Omega_1 n)]u[n]$	$\frac{1-z^{-1}\cos\Omega_1}{1-z^{-1}2\cos\Omega_1+z^{-2}}$	$\lvert z\rvert>1$
$[sen(\Omega_1 n)]u[n]$	$\frac{z^{-1}sen\Omega_1}{1-z^{-1}2\cos\Omega_1+z^{-2}}$	$\lvert z\rvert>1$
$[r^n\cos(\Omega_1 n)]u[n]$	$\frac{1-z^{-1}r\cos\Omega_1}{1-z^{-1}2r\cos\Omega_1+r^2z^{-2}}$	$\lvert z\rvert>r$
$[r^n sen(\Omega_1 n)]u[n]$	$\frac{z^{-1}r\,sen\Omega_1}{1-z^{-1}2r\cos\Omega_1+r^2z^{-2}}$	$\lvert z\rvert>r$

Transformadas bilaterales para señales que son distintas de cero para n<0

Señal	Transformada bilateral	ROC
$u[-n-1]$	$\frac{1}{1-z^{-1}}$	$\lvert z\rvert<1$
$-\alpha^n u[-n-1]$	$\frac{1}{1-\alpha z^{-1}}$	$\lvert z\rvert<\lvert\alpha\rvert$
$-n\alpha^n u[-n-1]$	$\frac{\alpha z^{-1}}{(1-\alpha z^{-1})^2}$	$\lvert z\rvert<\lvert\alpha\rvert$

Propiedades de la transformada z

Señal	Transformada unilateral	Transformada bilateral	ROC
x[n]	X(z)	X(z)	R_x
y[n]	Y(z)	Y(z)	R_y
ax[n]+by[n]	aX(z)+bY(z)	aX(z)+bY(z)	al menos $R_x \cap R_y$
x[n-k]	vea abajo	$z^{-k}X(z)$	R_x excepto posibl.para |z|=0,inf.
$\alpha^n x[n]$	$X(z/\alpha)$	$X(z/\alpha)$	$\|\alpha\| R_x$
x[-n]	no	X(1/z)	1/ R_x
x[n]*y[n]	X(z)Y(z)	X(z)Y(z)	al menos $R_x \cap R_y$
nx[n]	$-z\frac{d}{dz}X(z)$	$-z\frac{d}{dz}X(z)$	R_x excepto posibl. En la suma o eliminación de z=0

Propiedades de corrimiento en el tiempo de la transformada z unilateral

$$x[n-k] \xleftrightarrow{z_u} x[-k]+x[-k+1]z^{-1}+\cdots+x[-1]z^{-k+1}+\mathbf{z^{-k}X(z)}, \ \forall k>0$$

$$x[n-k] \xleftrightarrow{z_u} -x[0]z^{k}-x[1]z^{k-1}-\cdots-x[k-1]z+\mathbf{z^{k}X(z)}, \ \forall k>0$$

Problema 7.17d

7.17. Determine the z-transform and ROC for the following time signals: Sketch the ROC, poles, and zeros in the z-plane.

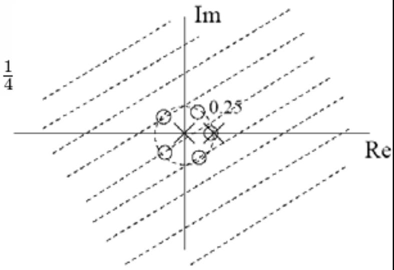
(d) $x[n] = \left(\frac{1}{4}\right)^n (u[n] - u[n-5])$

$$\begin{aligned} X(z) &= \sum_{n=0}^{4}\left(\frac{1}{4}z^{-1}\right)^n \\ &= \frac{1-\left(\frac{1}{4}z^{-1}\right)^5}{1-\frac{1}{4}z^{-1}} \\ &= \frac{\left[z^5-\left(\frac{1}{4}\right)^5\right]}{z^4\left(z-\frac{1}{4}\right)}, \quad \text{all } z \end{aligned}$$

4 poles at $z = 0$, 1 pole at $z = 0$

5 zeros at $z = \frac{1}{4}e^{jk\frac{2\pi}{5}}$ $k = 0, 1, 2, 3, 4$

Note zero for $k = 0$ cancels pole at $z = \frac{1}{4}$



Problema 7.17f

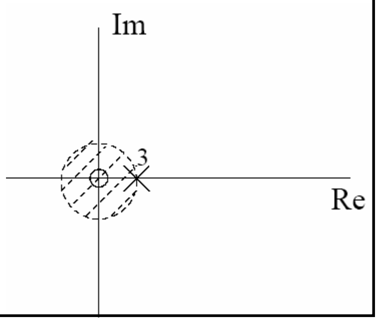
7.17. Determine the z-transform and ROC for the following time signals: Sketch the ROC, poles, and zeros in the z-plane. (f) $x[n] = 3^n u[-n-1]$

$$X(z) = \sum_{n=-\infty}^{-1} \left(3z^{-1}\right)^n = \sum_{n=1}^{\infty} \left(\frac{1}{3}z\right)^n$$

$$= \frac{\frac{1}{3}z}{1-\frac{1}{3}z} = \frac{-1}{1-3z^{-1}}, \quad |z| < 3$$

Pole at $z = 3$

Zero at $z = 0$



Problema 7.17g

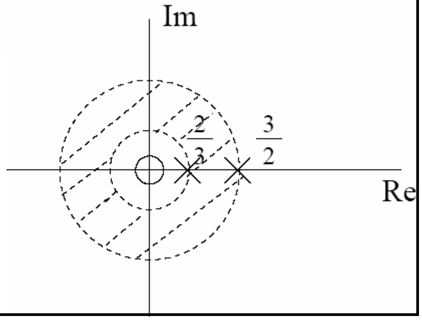
7.17. Determine the z-transform and ROC for the following time signals: Sketch the ROC, poles, and zeros in the z-plane.

(g) $x[n] = \left(\frac{2}{3}\right)^{|n|}$

$$X(z) = \sum_{n=-\infty}^{-1} \left(\frac{3}{2}z^{-1}\right)^n + \sum_{n=0}^{\infty} \left(\frac{2}{3}z^{-1}\right)^n$$

$$= \frac{-1}{1-\frac{3}{2}z^{-1}} + \frac{1}{1-\frac{2}{3}z^{-1}}$$

$$= \frac{-\frac{5}{6}z}{(z-\frac{3}{2})(z-\frac{2}{3})}, \quad \frac{2}{3} < |z| < \frac{3}{2}$$



Problema 7.18

7.18. Given the following z-transforms, determine whether the DTFT of the corresponding time signals exists without determining the time signal, and identify the DTFT in those cases where it exists:

(a) $X(z) = \frac{5}{1+\frac{1}{3}z^{-1}}, \quad |z| > \frac{1}{3}$

ROC includes $|z| = 1$, DTFT exists.

$$X(e^{j\Omega}) = \frac{5}{1+\frac{1}{3}e^{-j\Omega}}$$

(b) $X(z) = \frac{5}{1+\frac{1}{3}z^{-1}}, \quad |z| < \frac{1}{3}$

ROC does not include, $|z| = 1$, DTFT does not exist.

(c) $X(z) = \frac{z^{-1}}{(1-\frac{1}{2}z^{-1})(1+3z^{-1})}, \quad |z| < \frac{1}{2}$

ROC does not include, $|z| = 1$, DTFT does not exist.

(d) $X(z) = \frac{z^{-1}}{(1-\frac{1}{2}z^{-1})(1+3z^{-1})}, \quad \frac{1}{2} < |z| < 3$

ROC includes $|z| = 1$, DTFT exists.

$$X(e^{j\Omega}) = \frac{e^{-j\Omega}}{(1-\frac{1}{2}e^{-j\Omega})(1+3e^{-j\Omega})}$$

Problema 7.20b

7.20. Use the tables of z-transforms and z-transform properties given in Appendix E to determine the z-transforms of the following signals:

(b) $x[n] = n\left(\left(\frac{1}{2}\right)^n u[n] * \left(\frac{1}{4}\right)^n u[n-2]\right)$

$$a[n] = \left(\frac{1}{2}\right)^n u[n] \quad \stackrel{z}{\longleftrightarrow} \quad A(z) = \frac{1}{1-\frac{1}{2}z^{-1}}, \quad |z| > \frac{1}{2}$$

$$b[n] = \left(\frac{1}{4}\right)^n u[n] \quad \stackrel{z}{\longleftrightarrow} \quad B(z) = \frac{1}{1-\frac{1}{4}z^{-1}}, \quad |z| > \frac{1}{4}$$

$$c[n] = b[n-2] \quad \stackrel{z}{\longleftrightarrow} \quad C(z) = \frac{z^{-2}}{1-\frac{1}{4}z^{-1}}$$

$$x[n] = n[a[n] * b[n]] \quad \stackrel{z}{\longleftrightarrow} \quad X(z) = -z\frac{d}{dz}A(z)B(z)$$

$$X(z) = \frac{2z^{-2} - \frac{3}{4}z^{-3}}{1-\frac{3}{4}z^{-1}+\frac{1}{8}z^{-2})^2}, \quad |z| > \frac{1}{2}$$

Problema 7.20d

7.20. Use the tables of z-transforms and z-transform properties given in Appendix E to determine the z-transforms of the following signals:

(d) $x[n] = n\sin(\frac{\pi}{2}n)u[-n]$

$$\begin{aligned}
x[n] &= -n\sin(-\frac{\pi}{2}n)u[-n] \\
X(z) &= -z\frac{d}{dz}\left(\frac{z^{-1}}{1+z^{-2}}\right)\bigg|_{z=\frac{1}{z}} \\
&= -z\left[\frac{-z^{-2}}{1+z^{-2}} - \frac{z^{-1}(-2z^{-3})}{(1+z^{-2})^2}\right]\bigg|_{z=\frac{1}{z}} \\
&= \frac{z}{1+z^2} + \frac{2z^3}{(1+z^2)^2}
\end{aligned}$$

Problema 7.25

7.25. Determine the time-domain signals corresponding to the following z-transforms:

(a) $X(z) = 1 + 2z^{-6} + 4z^{-8}, \quad |z| > 0$

$$x[n] = \delta[n] + 2\delta[n-6] + 4\delta[n-8]$$

(b) $X(z) = \sum_{k=5}^{10} \frac{1}{k} z^{-k}, \quad |z| > 0$

$$x[n] = \sum_{k=5}^{10} \frac{1}{k}\delta[n-k]$$

(c) $X(z) = (1+z^{-1})^3, \quad |z| > 0$

$$\begin{aligned}
X(z) &= (\delta[n] + \delta[n-1]) * (\delta[n] + \delta[n-1]) * (\delta[n] + \delta[n-1]) \\
x[n] &= \delta[n] + 3\delta[n-1] + 3\delta[n-2] + \delta[n-3]
\end{aligned}$$

(d) $X(z) = z^6 + z^2 + 3 + 2z^{-3} + z^{-4}, \quad |z| > 0$

$$x[n] = \delta[n+6] + \delta[n+2] + 3\delta[n] + 2\delta[n-3] + \delta[n-4]$$

Problema 7.28

7.28. Use a power series expansion to determine the time-domain signal corresponding to the following z-transforms:

(a) $X(z) = \frac{1}{1-\frac{1}{4}z^{-2}}, \quad |z| > \frac{1}{4}$

$$X(z) = \sum_{k=0}^{\infty}(\frac{1}{4}z^{-2})^k$$

$$x[n] = \sum_{k=0}^{\infty}(\frac{1}{4})^k\delta[n-2k]$$
$$= \begin{cases} (\frac{1}{4})^{\frac{n}{2}} & n \text{ even and } n \geq 0 \\ 0 & n \text{ odd} \end{cases}$$

(b) $X(z) = \frac{1}{1-\frac{1}{4}z^{-2}}, \quad |z| < \frac{1}{4}$

$$X(z) = -4z^2\sum_{k=0}^{\infty}(2z)^{2k}$$
$$= -\sum_{k=0}^{\infty}2^{2(k+1)}z^{2(k+1)}$$
$$x[n] = -\sum_{k=0}^{\infty}2^{2(k+1)}\delta[n+2(k+1)]$$

Problema 7.34

7.34. Determine whether each of the systems described below are (i) causal and stable and (ii) minimum phase.

(a) $H(z) = \frac{2z+3}{z^2+z-\frac{5}{16}}$

zero at: $z = -\frac{3}{2}$

poles at: $z = -\frac{5}{4}, \frac{1}{4}$

(i) Not all poles are inside $|z| = 1$, the system is not causal and stable.

(ii) Not all poles and zeros are inside $|z| = 1$, the system is not minimum phase.

(b) $y[n] - y[n-1] - \frac{1}{4}y[n-2] = 3x[n] - 2x[n-1]$

zeros at: $z = 0, \frac{2}{3}$

poles at: $z = \frac{1 \pm \sqrt{2}}{2}$

(i) Not all poles are inside $|z| = 1$, the system is not causal and stable.

(ii) Not all poles and zeros are inside $|z| = 1$, the system is not minimum phase.

(c) $y[n] - 2y[n-2] = x[n] - \frac{1}{2}x[n-1]$

$$H(z) = \frac{z(z-\frac{1}{2})}{z^2-2}$$

zeros at: $z = 0, \frac{1}{2}$

poles at: $z = \pm\sqrt{2}$

(i) Not all poles are inside $|z| = 1$, the system is not causal and stable.

(ii) Not all poles and zeros are inside $|z| = 1$, the system is not minimum phase.

Problema 7.00

Se considera que la demanda (d), en el mercado de automóviles, aumenta/disminuye linealmente al disminuir/aumentar el precio (p) de los automóviles, ecuación (1), y que el número de automóviles que los constructores deciden fabricar (r) aumenta/ /disminuye linealmente al aumentar/disminuir el precio de los automóviles, ecuación (2)

$$d(kT) = d_0 - a\,p(kT) \quad (1)$$

$$r(kT) = r_0 + b\,p(kT) \quad (2)$$

siendo d_0, a, r_0 y b constantes positivas

Si se supone que el comportamiento de la demanda y la orden de producción de automóviles se renueva cada semana (periodo de muestreo), y que el tiempo medio en fabricar un automóvil (c) es de dos semanas, determinar :

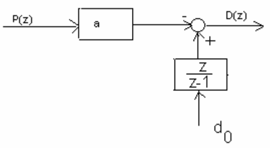
1. El esquema de bloques en z que relacione los automóviles fabricados en función del precio, y el esquema de bloques que relacione la demanda de automóviles en función del precio.
2. Si la demanda es igual al número de automóviles fabricados para cualquier instante de tiempo, deducir cuál es la ecuación en diferencias del precio. ¿Es estable el precio de los automóviles, y en qué condiciones?. Predecir el precio de los automóviles las próximas 5 semanas, si el precio actual es de 1.200.000 Ptas y la semana pasada era de 1.100.000 Ptas, tomando los siguientes valores numéricos:

d_0 = 2.400.000
r_0 = 1.300.000
a = 0,5
b = 0,4

Problema 7.00a

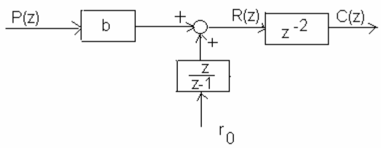
1)

$$d(kT) = d_0 - ap(kT) \xrightarrow{z} D(z) = \left(\frac{z}{z-1}\right)d_0 - aP(z)$$



$$r(kT) = r_0 + bp(kT) \xrightarrow{z} R(z) = \left(\frac{z}{z-1}\right)r_0 + bP(z)$$

$$c(kT) = r[(k-2)T] \xrightarrow{z} C(z) = z^{-2}R(z)$$



2) $d(kT) = c(kT) \quad (3)$

$$(1),(3) \Rightarrow d_0 - ap(kT) = r[(k-2)T] = r_0 + bp[(k-2)T]$$

$$ap(kT) + bp[(k-2)T] = d_0 - r_0 \xrightarrow{z} aP(z) + bz^{-2}P(z) = (d_0 - r_0)\frac{z}{z-1}$$

$$(a + bz^{-2})P(z) = (d_0 - r_0)\frac{z}{z-1}$$

$$P(z) = \frac{1}{a + bz^{-2}}(d_0 - r_0)\frac{z}{z-1} \quad \textit{independientemente de la señal de entrada la}$$

ecuación característica es $a + bz^{-2} = 0 \Rightarrow polos: z = \pm\sqrt{-\frac{a}{b}} = \pm\sqrt{\frac{a}{b}}\,j$

$b < a$ *sistema estable*

$b = a$ *sistema marginalmente estable*

Problema 7.00b

$$p(kT) = \frac{d_0 - r_0}{a} - \frac{b}{a} p\left[(k-2)T\right] = \frac{2.400.000 - 1.300.000}{0,5} - \frac{0,4}{0,5} p\left[(k-2)T\right]$$

$$p(kT) = 2.200.000 - 0.8\, p\left[(k-2)T\right]$$

$$p(-1T) = 1.100.000$$
$$p(0T) = 1.200.000$$
$$p(1T) = 2.200.000 - 0.8 * 1.100.000 = 1.320.000$$
$$p(2T) = 2.200.000 - 0.8 * 1.200.000 = 1.240.000$$
$$p(3T) = 2.200.000 - 0.8 * 1.320.000 = 1.144.000$$
$$p(4T) = 2.200.000 - 0.8 * 1.240.000 = 1.208.000$$
$$p(5T) = 2.200.000 - 0.8 * 1.144.000 = 1.284.800$$