CAPITULO 6.- LA TRANSFORMADA DE LAPLACE.

6.1 Introducción.

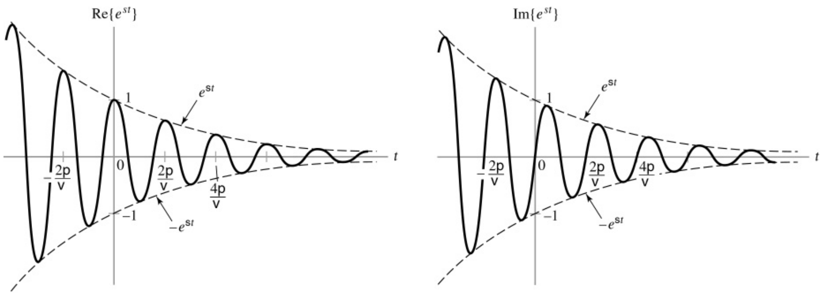
Generalizamos la representación senoidal compleja de la FT
Caracterización más amplia de sistemas y su interacción con señales
*señales no absolutamente sumables
(ej. : respuesta impulso de un sistema inestable)
Propiedades
Función de transferencia
Unilateral: solución ecuaciones diferenciales con condiciones iniciales
Bilateral: características del sistema (estabilidad, causalidad, resp. frec.)

Análisis transitorio y estabilidad de sistemas LTI

6.2 La transformada de Laplace

$$s = \sigma + j\omega$$

$$e^{st} = e^{\sigma t} e^{j\omega t} = e^{\sigma t}\cos(\omega t) + j e^{\sigma t} sen(\omega t)$$



$\sigma =$ Factor de amortiguamiento exponencial < 0

$\omega =$ Frecuencia del coseno y seno

6.2 a

(a) (b)

$$x(t) = e^{st} \quad \Rightarrow \quad y(t) = H\left\{e^{st}\right\} = h(t) * x(t) = \int_{-\infty}^{\infty} h(\tau)x(t-\tau)d\tau$$

$$y(t) = \int_{-\infty}^{\infty} h(\tau)e^{s(t-\tau)}d\tau = \left[\int_{-\infty}^{\infty} h(\tau)e^{-s\tau}d\tau\right]e^{st} = H(s)e^{st}$$

$$H\left\{e^{st}\right\} = H(s)e^{st} \quad ; \quad H(s) = \int_{-\infty}^{\infty} h(\tau)e^{-s\tau}d\tau$$

$e^{st} =$ *función característica del sistema*

$H(s) =$ *valor característico del sistema*

$H(s) = |H(s)|e^{j\phi(s)}$; $\phi(s) =$ *fase de* $H(s)$

$$y(t) = |H(s)|e^{j\phi(s)}e^{st} = |H(\sigma + j\omega)|e^{\sigma t}e^{j(\omega t + \phi(\sigma + j\omega))}$$

6.2 b

$$e^{st} = e^{\sigma t}\cos(\omega t) + je^{\sigma t} sen(\omega t)$$

$$y(t) = |H(\sigma + j\omega)| e^{\sigma t} \cos(\omega t + \phi(\sigma + j\omega)) + j|H(\sigma + j\omega)| e^{\sigma t} sen(\omega t + \phi(\sigma + j\omega))$$

$$H(s) = \int_{-\infty}^{\infty} h(\tau) e^{-s\tau} d\tau \quad ; \quad s = \sigma + j\omega$$

$$H(\sigma + j\omega) = \int_{-\infty}^{\infty} h(t) e^{-(\sigma + j\omega)t} dt = \int_{-\infty}^{\infty} \left[h(t) e^{-\sigma t}\right] e^{-j\omega t} d\tau$$

$$FT: \quad X(j\omega) = \int_{-\infty}^{\infty} [x(t)] e^{-j\omega t} dt \quad ; \quad [x(t)] = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\omega) e^{j\omega t} d\omega$$

$$h(t) e^{-\sigma t} \xleftrightarrow{FT} H(\sigma + j\omega)$$

$$h(t) e^{-\sigma t} = \frac{1}{2\pi} \int_{-\infty}^{\infty} H(\sigma + j\omega) e^{j\omega t} d\omega$$

$$h(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} H(\sigma + j\omega) e^{(\sigma + j\omega)t} d\omega$$

6.2 c

$$h(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} H(\sigma + j\omega) e^{(\sigma + j\omega)t} d\omega \quad ; \quad s = \sigma + j\omega \quad ; \quad d\omega = \frac{ds}{j}$$

$$h(t) = \frac{1}{2\pi j} \int_{\sigma - j\infty}^{\sigma + j\infty} H(s) e^{st} ds$$

$$H(s) = \int_{-\infty}^{\infty} h(t) e^{-st} dt \qquad ; \qquad Para\ toda\ señal\ x(t)$$

$$x(t) = \frac{1}{2\pi j} \int_{\sigma - j\infty}^{\sigma + j\infty} X(s) e^{st} ds = Transformada\ inversa\ de\ Laplace\ de\ H(s)$$

$$X(s) = \int_{-\infty}^{\infty} x(t) e^{-st} dt \quad = Transformada\ de\ Laplace\ de\ h(t)$$

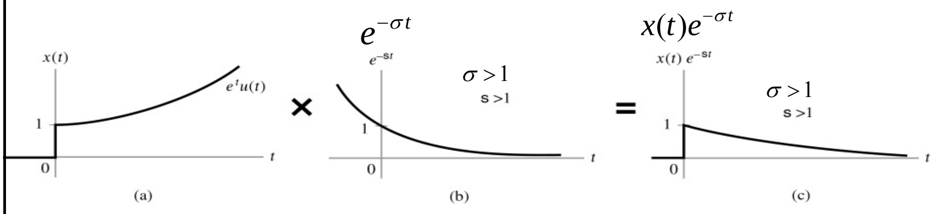
$$x(t) \xleftrightarrow{L} X(s) \qquad ; \qquad x(t) e^{-\sigma t} \xleftrightarrow{FT} X(\sigma + j\omega) = X(s)$$

$$condición\ de\ convergencia \quad ; \quad \int_{-\infty}^{\infty} \left| x(t) e^{-st} \right| dt < \infty$$

ROC : **Región de convergencia**=intervalo de σ para converja transfor.

Figure 6.2 (p. 485)

The Laplace transform applies to more general signals than the Fourier transform does. (a) Signal for which the Fourier transform does not exist.
(b) Attenuating factor associated with Laplace transform. (c) The modified signal $x(t)e^{-\sigma t}$ is absolutely integrable for $\sigma > 1$.



6.2 d

Si x(t) es absolutamente sumable: $X(j\omega) = X(s)|_{\sigma=0}$

La transformada de Fourier está dada por la transformada de Laplace evaluada a lo largo del eje imaginario

$$X(S) = \frac{b_M s^M + b_{M-1}s^{M-1} + \ldots + b_1 s + b_0}{s^N + a_{N-1}s^{N-1} + \ldots + a_1 s + a_0} = \frac{b_M \prod_{k=1}^{M}(s - c_k)}{\prod_{k=1}^{N}(s - d_k)}$$

$$c_k : \quad ceros\ de\ X(S) \quad \Rightarrow \quad o$$

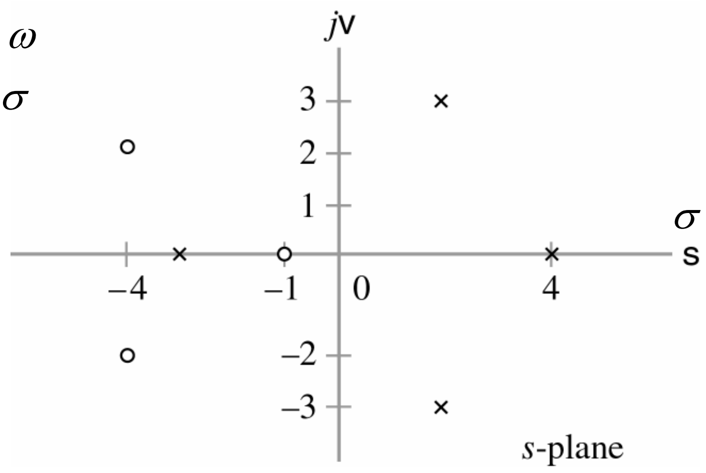
$$d_k : \quad polos\ de\ X(S) \quad \Rightarrow \quad x$$

Figure 6.3 (p. 486)

The *s*-plane. The horizontal axis is Re{*s*} and the vertical axis is Im{*s*}. Zeros are depicted at *s* = –1 and *s* = –4 ± 2*j*, and poles are depicted at
s = –3, *s* = 2 ± 3*j*, and *s* = 4.

$\mathrm{v} \Rightarrow \omega$

$\mathrm{s} \Rightarrow \sigma$



6.2 e

Ejemplo 6.1. Determine la transformada de Laplace de

$$x(t) = e^{at}u(t) \qquad \text{(causal)}$$

y describa la ROC y la localización de polos y ceros. Suponer "a" real

$$X(s) = \int_{-\infty}^{\infty} e^{at}u(t)e^{-st}dt = \int_{0}^{\infty} e^{-(s-a)t}dt = \frac{-1}{s-a}e^{-(s-a)t}\Big|_0^{\infty}$$

$$\lim_{t\to\infty} e^{-(s-a)t} = \lim_{t\to\infty} e^{-(\sigma-a)t}e^{-j\omega t} \overset{\sigma>a}{=} 0$$

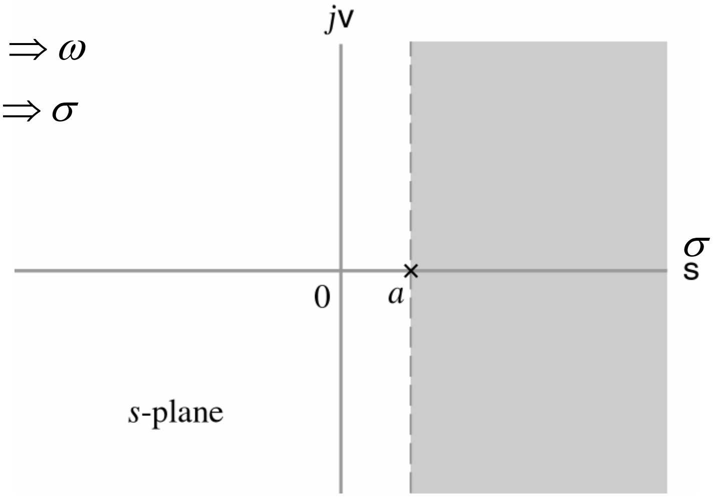
$$X(s) = \frac{1}{s-a} \quad , \quad \mathrm{Re}(s) > a \quad \Rightarrow ROC$$

$$polo \quad s = a$$

Figure 6.4 (p. 487)
The ROC for $x(t) = e^{at}u(t)$ is depicted by the shaded region. A pole is located at $s = a$.

$v \Rightarrow \omega$

$s \Rightarrow \sigma$



6.2 f

Ejemplo 6.1. Determine la transformada de Laplace de :

$$y(t) = -e^{at}u(-t) \qquad \text{(anticausal)}$$

y describa la ROC y la localización de polos y ceros. Suponer "a" real

$$Y(s) = \int_{-\infty}^{\infty} \left[-e^{at}u(-t) \right] e^{-st} dt = -\int_{-\infty}^{0} e^{-(s-a)t} dt = \frac{1}{s-a} e^{-(s-a)t} \Big|_{-\infty}^{0}$$

$$\lim_{t\to-\infty} e^{-(s-a)t} = \lim_{t\to-\infty} e^{-(\sigma-a)t} e^{-j\omega t} \overset{\sigma<a}{=} 0$$

$$X(s) = \frac{1}{s-a} \quad , \quad \text{Re}(s) < a \quad \Rightarrow ROC$$

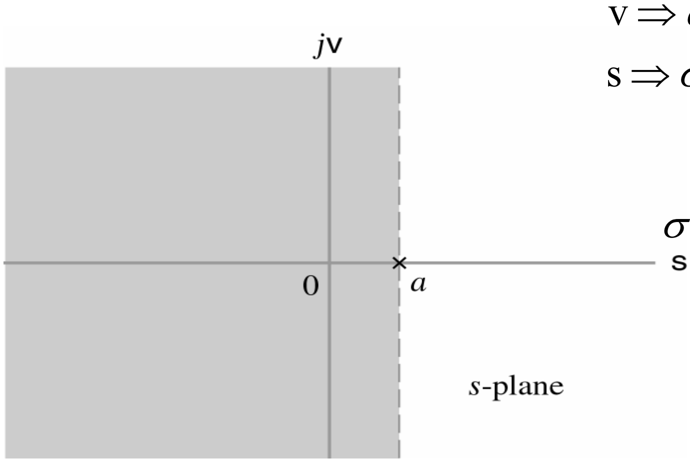
$$polo \quad s = a$$

$$Y(s) = X(s)$$

Figure 6.5 (p. 488)
The ROC for $y(t) = -e^{at}u(-t)$ is depicted by the shaded region.
A pole is located at $s = a$.

$v \Rightarrow \omega$

$s \Rightarrow \sigma$



6.3 La transformada de Laplace unilateral.

$$X(s) = \int_{0^+}^{\infty} x(t)e^{-st}dt \quad ; \quad x(t) \xleftrightarrow{L_u} X(s)$$

$$e^{at}u(t) \xleftrightarrow{L_u} \frac{1}{s-a} \quad ; \quad e^{at}u(t) \xleftrightarrow{L} \frac{1}{s-a} \quad ROC \quad \text{Re}(s) > a$$

PROPIEDADES $x(t) \xleftrightarrow{L_u} X(s) \quad ; \quad y(t) \xleftrightarrow{L_u} Y(s)$

Linealidad $ax(t) + by(t) \xleftrightarrow{L_u} aX(s) + bY(s)$

Escalamiento $x(at) \xleftrightarrow{L_u} \frac{1}{|a|} X\left(\frac{s}{a}\right)$

Corrimiento en el tiempo $x(t-\tau) \xleftrightarrow{L_u} e^{-s\tau} X(s)$

$$\forall \tau \mid x(t-\tau)u(t) = x(t-\tau)u(t-\tau)$$

PROPIEDADES (cont.)

6.3 a

Corrimiento en el dominio s $e^{s_0 t}x(t) \xleftrightarrow{L_u} X(s-s_0)$

Convolución $x(t) * y(t) \xleftrightarrow{L_u} X(s)Y(s)$

Diferenciación en el dominio de s $-tx(t) \xleftrightarrow{L_u} \frac{d}{ds}X(s)$

Diferenciación en el dominio del tiempo $\frac{d}{dt}x(t) \xleftrightarrow{L_u} sX(s) - x(0^+)$

$$\frac{d^n}{dt^n}x(t) \xleftrightarrow{L_u} s^n X(s) - \frac{d^{n-1}}{dt^{n-1}}x(t)|_{t=0^+} - \ldots - s^{n-2}\frac{d}{dt}x(t)|_{t=0^+} - s^{n-1}x(0^+)$$

Integración $\int_{-\infty}^{t} x(\tau)d\tau \xleftrightarrow{L_u} \frac{1}{s}\int_{-\infty}^{0^+} x(\tau)d\tau + \frac{X(s)}{s}$

Teorema valor inicial $\lim_{s\to\infty} sX(s) = x(0^+)$

Teorema valor final $\lim_{s\to 0} sX(s) = x(\infty)$ solo si Re(polos)<0

6.4 Inversión de la transformada de Laplace.

$$x(t) = \frac{1}{2\pi}\int_{\sigma-j\infty}^{\sigma+j\infty} X(s)e^{st}ds$$

$$X(S) = \frac{B(s)}{A(s)} = \frac{b_M s^M + b_{M-1}s^{M-1} + \ldots + b_1 s + b_0}{s^N + a_{N-1}s^{N-1} + \ldots + a_1 s + a_0}$$

$$X(S) = \frac{b_M s^M + b_{M-1}s^{M-1} + \ldots + b_1 s + b_0}{\prod_{k=1}^{N}(s-d_k)}$$

<u>Polos reales:</u>

polos d_k distintos, $X(S) = \sum_{k=1}^{N}\frac{A_k}{s-d_k}$; $A_k e^{d_k t}u(t) \xleftrightarrow{L_u} \frac{A_k}{s-d_k}$

polo d_i multiplicidad r, $\frac{A_{i_1}}{s-d_i}, \frac{A_{i_2}}{(s-d_i)^2}, \ldots, \frac{A_{i_r}}{(s-d_i)^r}$

$$\frac{At^{n-1}}{(n-1)!}e^{d_k t}u(t) \xleftrightarrow{L_u} \frac{A}{(s-d_i)^n}$$

6.4 a

Polos complejos

Si los coeficientes en el polinomio del denominador son reales, los polos complejos se presentan en pares conjugados complejos.

$$polos\ conjugados\ complejos: \quad \alpha + j\omega_0 \quad , \quad \alpha - j\omega_0$$

$$\frac{A_1}{s-\alpha-j\omega_0}+\frac{A_1}{s-\alpha+j\omega_0}=\frac{B_1 s+B_2}{(s-\alpha-j\omega_0)(s-\alpha+j\omega_0)}=\frac{B_1 s+B_2}{(s-\alpha)^2+\omega_0{}^2}=$$

$$=\frac{C_1(s-\alpha)}{(s-\alpha)^2+\omega_0{}^2}+\frac{C_2\omega_0{}^2}{(s-\alpha)^2+\omega_0{}^2} \quad ; \quad C_1=B_1 \quad ; \quad C_2=\frac{B_1\alpha+B_2}{\omega_0{}^2}$$

$$C_1 e^{\alpha t}\cos(\omega_0 t)u(t) \xleftrightarrow{L_u} \frac{C_1(s-\alpha)}{(s-\alpha)^2+\omega_0{}^2}$$

$$C_2 e^{\alpha t} sen(\omega_0 t)u(t) \xleftrightarrow{L_u} \frac{C_2\omega_0{}^2}{(s-\alpha)^2+\omega_0{}^2}$$

Transformadas de Laplace básicas (1)

Señal	Transformada	ROC
$x(t)=\frac{1}{2\pi j}\int_{\sigma-j\infty}^{\sigma+j\infty}X(s)e^{st}ds$	$X(s)=\int_{-\infty}^{\infty}x(t)e^{-st}dt$	
u(t)	$\frac{1}{s}$	Re{s}>0
t u(t)	$\frac{1}{s^2}$	Re{s}>0
$\delta(t-\tau), \tau \geq 0$	e^{-st}	$\forall\ s$
$e^{-at}u(t)$	$\frac{1}{s+a}$	Re{s}>-a
$te^{-at}u(t)$	$\frac{1}{(s+a)^2}$	Re{s}>-a

Transformadas de Laplace básicas (2)

Señal	Transformada	ROC
$x(t)=\frac{1}{2\pi j}\int_{\sigma-j\infty}^{\sigma+j\infty}X(s)e^{st}ds$	$X(s)=\int_{-\infty}^{\infty}x(t)e^{-st}dt$	
$[\cos(\omega_1 t)]u(t)$	$\frac{s}{s^2+\omega_1^2}$	Re{s}>0
$[sen(\omega_1 t)]u(t)$	$\frac{\omega_1}{s^2+\omega_1^2}$	Re{s}>0
$[e^{-at}\cos(\omega_1 t)]u(t)$	$\frac{s+a}{(s+a)^2+\omega_1^2}$	Re{s}>-a
$[e^{-at}sen(\omega_1 t)]u(t)$	$\frac{\omega_1}{(s+a)^2+\omega_1^2}$	Re{s}>-a
$t^n u(t)$	$\frac{n!}{s^{n+1}}$	Re{s}>0

6.4 b

Problema 6.8

Encontrar la transformada inversa de Laplace de $X(s)=\frac{3s+2}{s^2+4s+5}$

$$X(s)=\frac{3s+2}{s^2+4s+5}=\frac{3s+2}{(s+2)^2+1^2}=\frac{B_1 s+B_2}{(s-\alpha)^2+\omega_0{}^2};\quad B_1=3\,;B_2=2$$

$$X(s)=\frac{C_1(s-\alpha)}{(s-\alpha)^2+\omega_0{}^2}+\frac{C_2\omega_0{}^2}{(s-\alpha)^2+\omega_0{}^2};C_1=B_1=3\,;C_2=\frac{B_1\alpha+B_2}{\omega_0{}^2}=-4$$

$$X(s)=\frac{3(s+2)}{(s+2)^2+1^2}+\frac{-4\;1^2}{(s+2)^2+1^2}$$

$$C_1e^{\alpha t}\cos(\omega_0 t)u(t)\xleftrightarrow{L_u}\frac{C_1(s-\alpha)}{(s-\alpha)^2+\omega_0{}^2}$$

$$C_2e^{\alpha t}sen(\omega_0 t)u(t)\xleftrightarrow{L_u}\frac{C_2\omega_0{}^2}{(s-\alpha)^2+\omega_0{}^2}$$

$$x(t)=3e^{-2t}\cos(t)u(t)-4e^{-2t}sen(t)u(t)$$

6.5 Solución de ecuaciones diferenciales con condiciones iniciales.

$$a_N \frac{d^N}{dt^N} y(t) + a_{N-1} \frac{d^{N-1}}{dt^{N-1}} y(t) + \ldots + a_1 \frac{d}{dt} y(t) + a_0 y(t) =$$

$$= b_M \frac{d^M}{dt^M} x(t) + b_{M-1} \frac{d^{M-1}}{dt^{M-1}} x(t) + \ldots + b_1 \frac{d}{dt} x(t) + b_0 x(t)$$

....

$$A(s) = a_N s^N + a_{N-1} s^{N-1} + \ldots + a_1 s + a_0$$

$$B(s) = b_M s^M + b_{M-1} s^{M-1} + \ldots + b_1 s + b_0$$

$$C(s) = \sum_{k=1}^{N} \sum_{l=0}^{k-1} a_k s^{k-1-l} \frac{d^l}{dt^l} y(t)|_{t=0^+}$$

$$D(s) = \sum_{k=1}^{M} \sum_{l=0}^{k-1} b_k s^{k-1-l} \frac{d^l}{dt^l} x(t)|_{t=0^+}$$

$$A(s)Y(s) - C(s) = B(s)X(s) - D(s)$$

6.5 a

$$A(s)Y(s) - C(s) = B(s)X(s) - D(s)$$

$$Y(s) = \frac{B(s)X(s) - D(s)}{A(s)} + \frac{C(s)}{A(s)}$$

$\textit{condiciones iniciales nulas} \Rightarrow C(s) = 0 \Rightarrow Y(s) = Y^{(f)}(s)$

$\textit{entrada nula} \quad \Rightarrow B(s)X(s) - D(s) = 0 \Rightarrow Y(s) = Y^{(n)}(s)$

$$Y^{(f)}(s) = \frac{B(s)X(s) - D(s)}{A(s)}$$

$Y^{(n)}(s) = \dfrac{C(s)}{A(s)} \quad ; \quad A(s) = 0 \Rightarrow \textit{raices} = p_i \; ; \quad y(t) = \sum_i e^{p_i t}$

p_i = frecuencias naturales del sistema

$$Y(s) = Y^{(f)}(s) + Y^{(n)}(s)$$

Frecuencias naturales del sistema : raíces de A(s)=0

Problema 6.38

Problema 6.38 Determine la respuesta natural y forzada para el sistema LTI descrito por la siguiente ecuación diferencial

$$\frac{d}{dt}y(t) + 10y(t) = 10x(t), \quad y(0^-) = 1, \quad x(t) = u(t)$$

$$\begin{aligned}
X(s) &= \frac{1}{s} \\
Y(s)(s+10) &= 10X(s) + y(0^-) \\
Y^f(s) &= \frac{10X(s)}{s+10} \\
&= \frac{10}{s(s+10)} \\
&= \frac{1}{s} + \frac{-1}{s+10} \\
y^f(t) &= \left[1 - e^{-10t}\right] u(t) \\
Y^n(s) &= \frac{y(0^-)}{s+10} \\
y^n(t) &= e^{-10t}u(t)
\end{aligned}$$

6.6 La transformada de Laplace bilateral.

$$x(t) \xleftrightarrow{L} X(s) = \int_{-\infty}^{\infty} x(t)e^{-st}dt \quad ; \quad x(t) \text{ no causal}$$

<u>Propiedades de la región de convergencia ROC</u>

Si x(t) es de orden exponencial.

La ROC de una señal lateral izquierda es de la forma $\sigma<\sigma_n$

$[x(t) = 0, \forall t > b]$

La ROC de una señal lateral derecha es de la forma $\sigma>\sigma_n$

$[x(t) = 0, \forall t < a]$

La ROC de una señal bilateral es de la forma $\sigma_p<\sigma<\sigma_n$

(es de extensión infinita en ambas direcciones)

Propiedades de la región de convergencia ROC 6.6 a

Las propiedades de: linealidad, escalamiento,corrimiento, convolución y diferenciación en el dominio s son idénticas para la transformada bilateral y unilateral, aunque las operaciones asociadas con estas propiedades pueden cambiar la ROC.

Corrimiento en el tiempo $x(t-\tau) \stackrel{L}{\longleftrightarrow} e^{-st} X(s)$

Diferenciación en el dominio del tiempo $\frac{d}{dt} x(t) \stackrel{L}{\longleftrightarrow} sX(s)$

con ROC al menos R_x

Integración en el tiempo

$$\int_{-\infty}^{t} x(\tau) d\tau \stackrel{L}{\longleftrightarrow} \frac{X(s)}{s}, \; ROC: R_x \cap \operatorname{Re}(s) > 0$$

Teoremas del valor inicial y final ,con la restricción x(t)=0 t<0

Inversión de la tranformada de Laplace bilateral 6.6 b

$$X(S) = \frac{b_M s^M + b_{M-1} s^{M-1} + \ldots + b_1 s + b_0}{s^N + a_{N-1} s^{N-1} + \ldots + a_1 s + a_0} = \frac{b_M \prod_{k=1}^{M} (s - c_k)}{\prod_{k=1}^{N} (s - d_k)} = \sum_{k=1}^{N} \frac{A_k}{s - d_k} \quad ;$$

$$A_k e^{d_k t} u(t) \stackrel{L}{\longleftrightarrow} \frac{A_k}{s - d_k} \quad con\ ROC\ \operatorname{Re}(s) > d_k \quad \text{Transf. lateral derecha}$$

$$-A_k e^{d_k t} u(-t) \stackrel{L}{\longleftrightarrow} \frac{A_k}{s - d_k} \quad con\ ROC\ \operatorname{Re}(s) < d_k \quad \text{Transf. lateral izquierda}$$

La ROC asociada a X(s) determina cual de las dos se elige.
La propiedad de linealidad establece que la ROC de X(s) es la Intersección de las ROC de los términos individuales en la expansión en fracciones parciales.
Si la señal es causal elegimos transformada lateral derecha.
Una señal estable es absolutamente integrable y existe la transformada de Fourier, la ROC incluye el eje Re(s)=0.

6.6 c

Problema 6.43 Use el método de la la descomposición en fracciones simples para calcular la señal en tiempo continuo correspondiente a la siguiente transformada bilateral de Laplace :

$$X(S) = \frac{-s-4}{s^2+3s+2}$$

$a)$ *Con ROC* $\operatorname{Re}(s) < -2$

$b)$ *Con ROC* $\operatorname{Re}(s) > -1$

$c)$ *Con ROC* $-2 < \operatorname{Re}(s) < -1$

$$X(S) = \frac{-s-4}{s^2+3s+2} = \frac{-3}{s+1} + \frac{2}{s+2}$$

$a) \quad x(t) = (3e^{-t} - 2e^{-2t})u(-t)$

$b) \quad x(t) = (-3e^{-t} + 2e^{-2t})u(t)$

$c) \quad x(t) = -3e^{-t}u(-t) + 2e^{-2t}u(t)$

6.7 Análisis de sistemas mediante la transformada de Laplace.

(a) (b)

$$y(t) = h(t) * x(t) \overset{L}{\longleftrightarrow} Y(s) = H(s)X(s) \Rightarrow H(s) = \frac{Y(s)}{X(s)} =$$ Función de transferencia

6.7 a

Función de transferencia y las ecuaciones diferenciales

$$\sum_{k=0}^{N} a_k \frac{d^k}{dt^k} y(t) = \sum_{k=0}^{M} b_k \frac{d^k}{dt^k} x(t) \xleftrightarrow{L} \sum_{k=0}^{N} a_k s^k Y(s) = \sum_{k=0}^{M} b_k s^k X(s)$$

$$x(t) = e^{st} \Rightarrow y(t) = H(s) e^{st}$$

$$\sum_{k=0}^{N} a_k \frac{d^k}{dt^k} \{e^{st}\} H(s) = \sum_{k=0}^{M} b_k \frac{d^k}{dt^k} \{e^{st}\} \quad ; \quad \frac{d^k}{dt^k} \{e^{st}\} = s^k e^{st}$$

$$H(s) = \frac{Y(s)}{X(s)} = \frac{\sum_{k=0}^{M} b_k s^k}{\sum_{k=0}^{N} a_k s^k} = \frac{b_M}{a_N} \frac{\prod_{k=1}^{M} (s - c_k)}{\prod_{k=!}^{N} (s - d_k)}$$

6.7 b

Descripción de función de transferencia y variables de estado

$$\mathbf{q}(t) = \begin{bmatrix} q_1(t) \\ q_2(t) \\ \vdots \\ q_N(t) \end{bmatrix} \xleftrightarrow{L} \tilde{\mathbf{q}}(s) = \begin{bmatrix} Q_1(s) \\ Q_2(s) \\ \vdots \\ Q_N(s) \end{bmatrix}$$

$$\frac{d}{dt} \mathbf{q}(t) = \mathbf{A}\mathbf{q}(t) + \mathbf{b}x(t) \xleftrightarrow{L} s\tilde{\mathbf{q}}(s) - \mathbf{A}\tilde{\mathbf{q}}(s) = \mathbf{b}X(s)$$

$$y(t) = \mathbf{c}\mathbf{q}(t) + dx(t) \xleftrightarrow{L} Y(s) = \mathbf{c}\tilde{\mathbf{q}}(s) + dX(s)$$

$$(s\mathbf{I} - \mathbf{A})\tilde{\mathbf{q}}(s) = \mathbf{b}X(s)$$

$$\tilde{\mathbf{q}}(s) = (s\mathbf{I} - \mathbf{A})^{-1}\mathbf{b}X(s)$$

$$Y(s) = \left[\mathbf{c}(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{b} + d\right] X(s) = H(s)X(s)$$

$$H(s) = \mathbf{c}(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{b} + d$$

Causalidad y estabilidad **6.7 c**

La respuesta al impulso es la transformada inversa de Laplace de la función de transferencia (¿ROC ?)
La descripción de la ecuación diferencial no da información de ROC

Si el sistema es causal h(t)=0 para t<0, la respuesta al impulso se determina de la función de transferencia empleando transformadas inversas de Laplace laterales derechas

Si el sistema es estable la respuesta al impulso es absolutamente integrable, existe la transformada de Fourier, la ROC incluye el eje jω en el plano s

Un sistema estable y causal deben tener todos los polos en el semiplano izquierdo del plano s

Figure 6.19 (p. 524)
The relationship between the locations of poles and the impulse response in a causal system. (a) A pole in the left half of the *s*-plane corresponds to an exponentially decaying impulse response. (b) A pole in the right half of the *s*-plane corresponds to an exponentially increasing impulse response. The system is unstable in this case.

$v \Rightarrow \omega$

$s \Rightarrow \sigma$

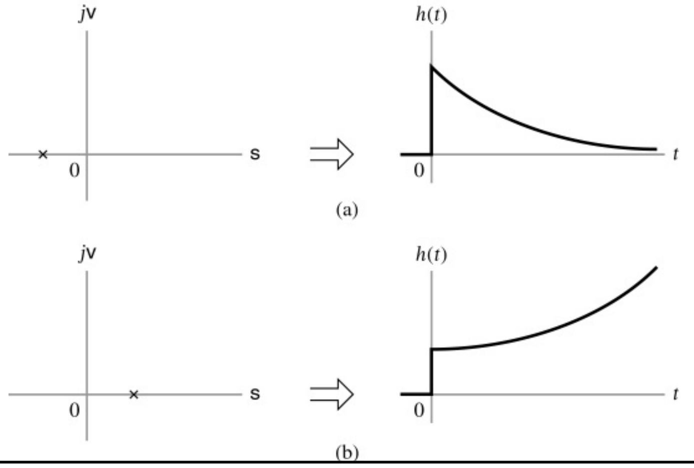
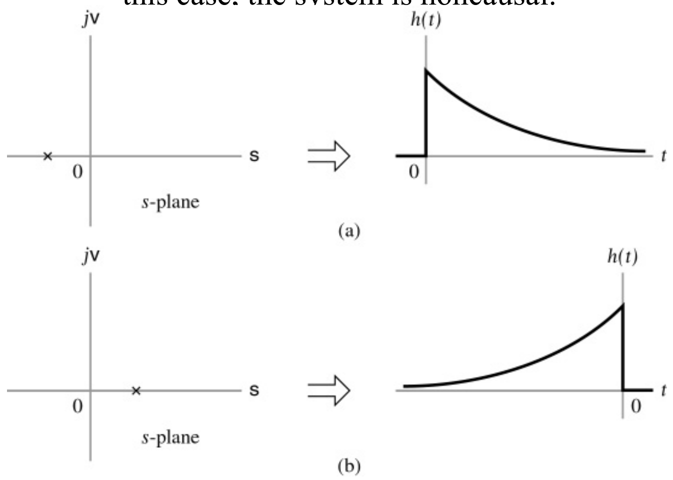


Figure 6.20 (p. 524)

The relationship between the locations of poles and the impulse response in a stable system. (a) A pole in the left half of the *s*-plane corresponds to a right-sided impulse response. (b) A pole in the right half of the *s*-plane corresponds to an left-sided impulse response. In this case, the system is noncausal.



v ⇒ ω

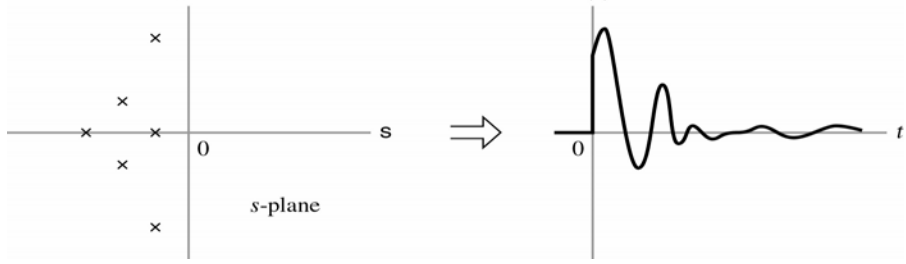
s ⇒ σ

Figure 6.21 (p. 525)

A system that is both stable and causal must have a transfer function with all of its poles in the left half of the *s*-plane, as shown here.

v ⇒ ω

s ⇒ σ



6.7 d

Determinación de la respuesta en frecuencia a partir de polos y ceros. Diagramas de Bode

$$H(s)=\frac{Y(s)}{X(s)}=\frac{\sum_{k=0}^{M} b_k s^k}{\sum_{k=0}^{N} a_k s^k}=\frac{b_M}{a_N}\frac{\prod_{k=1}^{M}(s-c_k)}{\prod_{k=!}^{N}(s-d_k)}$$

$$H(s)\xleftrightarrow{s=j\omega}H(j\omega)=\frac{b_M}{a_N}\frac{\prod_{k=1}^{M}(j\omega-c_k)}{\prod_{k=!}^{N}(j\omega-d_k)}=K\frac{\prod_{k=1}^{M}\left(1-\frac{j\omega}{c_k}\right)}{\prod_{k=1}^{N}\left(1-\frac{j\omega}{d_k}\right)}$$

$$k=\frac{b_M}{a_N}\frac{\prod_{k=1}^{M}(-c_k)}{\prod_{k=!}^{N}(-d_k)}$$

Calcularemos la respuesta en frecuencia del sistema combinando apropiadamente las respuestas en frecuencia de cada polo y cero. Suponemos que todos los **polos y ceros son reales**

6.7 e

$$H(j\omega)=K\frac{\prod_{k=1}^{M}\left(1-\frac{j\omega}{c_k}\right)}{\prod_{k=1}^{N}\left(1-\frac{j\omega}{d_k}\right)}$$

$$\left|H(j\omega)\right|_{db}=20\log|K|+\sum_{k=1}^{M}20\log\left|1-\frac{j\omega}{c_k}\right|-\sum_{k=1}^{M}20\log\left|1-\frac{j\omega}{d_k}\right| \qquad \text{sumas}$$

$$\arg\{H(j\omega)\}=\arg K+\sum_{k=1}^{M}\arg\left(1-\frac{j\omega}{c_k}\right)-\sum_{k=1}^{N}\arg\left(1-\frac{j\omega}{d_k}\right) \qquad \text{sumas}$$

Consideremos un factor polo

$$H_{d_k}(j\omega)=\frac{1}{1-\frac{j\omega}{d_k}} \quad ; \quad donde\ d_k=-\omega_b\ real$$

$$\left|H_{d_k}(j\omega)\right|_{db}=-20\log\left|1+\frac{j\omega}{\omega_b}\right|=-10\log\left(1+\frac{\omega^2}{\omega_b^2}\right)$$

6.7 e2

polo $\left|H_{d_k}(j\omega)\right|_{db} = -10\log\left(1+\dfrac{\omega^2}{\omega_b^2}\right)$

Asintotas bajas frecuencias, $\omega << \omega_b$

$$-10\log\left(1+\frac{\omega^2}{\omega_b^2}\right) \approx -10\log 1 = 0\ db$$

Asintotas altas frecuencias, $\omega >> \omega_b$

$$-10\log\left(1+\frac{\omega^2}{\omega_b^2}\right) \approx -10\log\left[\frac{\omega}{\omega_b}\right]^2 = -20\log\left[\frac{\omega}{\omega_b}\right] = -20\log|\omega| + 20\log|\omega_b|$$

corte de asintotas : $\omega = \omega_b \Rightarrow$ frecuencia de cruce o transición

6.7 f

$$\arg\{H(j\omega)\} = \arg K + \sum_{k=1}^{M}\arg\left(1-\frac{j\omega}{c_k}\right) - \sum_{k=1}^{N}\arg\left(1-\frac{j\omega}{d_k}\right)$$

un polo $H_{d_k}(j\omega) = \dfrac{1}{1-\dfrac{j\omega}{d_k}}$; *donde* $d_k = -\omega_b$ *real*

$$\arg\{H_{d_k}(j\omega)\} = -\arg\{1+j\omega/\omega_b\} = -\arctan\left(\frac{\omega}{\omega_b}\right)$$

Asintotas bajas frecuencias, $\omega << \omega_b$

$$-\arctan\left(\frac{\omega}{\omega_b}\right) \approx -\arctan(0) = 0$$

Asintotas bajas frecuencias, $\omega >> \omega_b$

$$-\arctan\left(\frac{\omega}{\omega_b}\right) \approx -\arctan(\infty) = -90^{o}$$

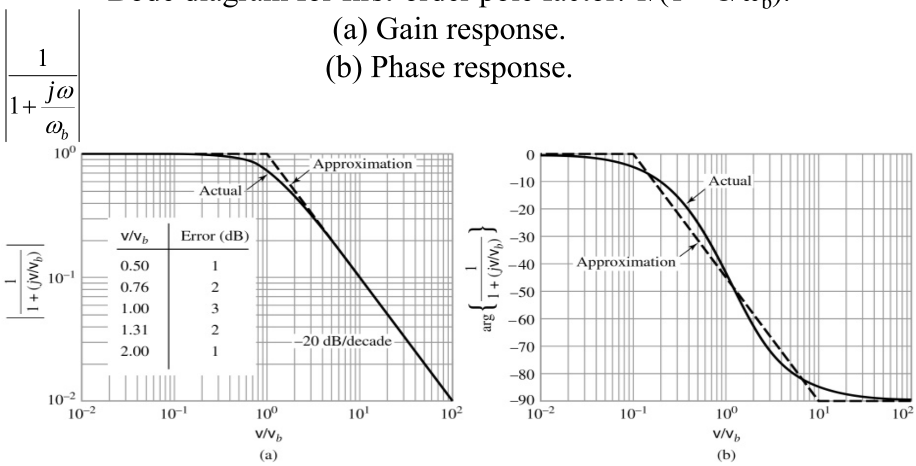
$$\omega = \omega_b \Rightarrow \arg\{H_{d_k}(j\omega_b)\} = -\arctan\left(\frac{\omega_b}{\omega_b}\right) = -45^{o}$$

Figure 6.30 (p. 535)

Bode diagram for first-order pole factor: $1/(1 + s/\omega_b)$.

(a) Gain response.

(b) Phase response.



ω / ω_b $\qquad$ ω / ω_b

$\mathrm{v} \Rightarrow \omega$

$\mathrm{s} \Rightarrow \sigma$

6.7 g

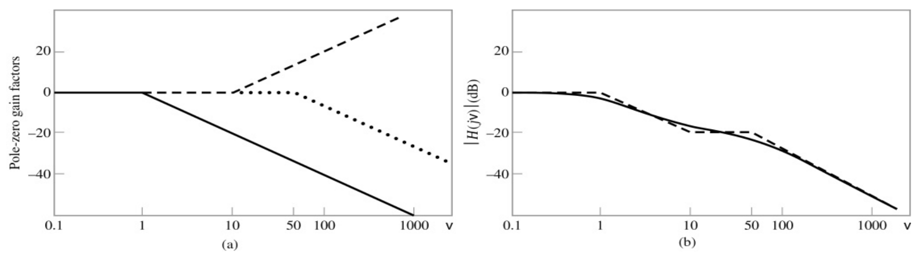
Sistema de fase mínima : “ todos los polos y ceros están en el semiplano izquierdo”

Ejemplo 6.25 Dibujar el diagrama de Bode de un sistema con función de transferencia

$$H(s) = \frac{5(s+10)}{(s+1)(s+50)}$$

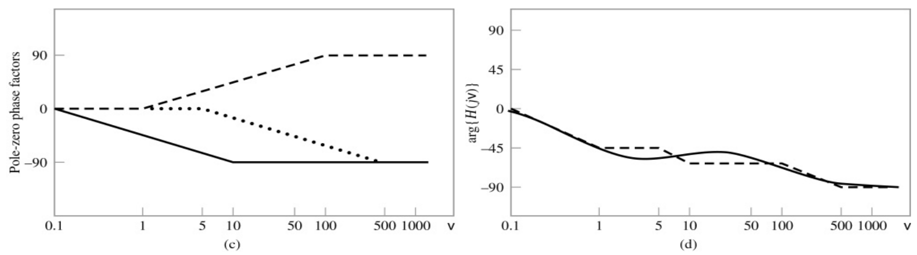
$$H(j\omega) = \frac{1 + \frac{j\omega}{10}}{(1+j\omega)\left(1 + \frac{j\omega}{50}\right)}$$

Figure 6.31a (p. 536)
Bode diagram for Example 6.25.
(a) Gain response of pole at $s = -1$ (solid line), zero at $s = -10$ (dashed line), and pole at $s = -50$ (dotted line).
(b) Actual gain response (solid line) and asymptotic approximation (dashed line).



$\text{v} \Rightarrow \omega$

Figure 6.31b (p. 536)
(c) Phase response of pole at $s = -1$ (solid line), zero at $s = -10$ (dashed line), and pole at $s = -50$ (dotted line).
(d) Actual phase response (solid line) and asymptotic approximation (dashed line).



$\text{v} \Rightarrow \omega$

Pares de **polos o ceros complejos conjugados** **6.7 h**

$$P(s)=\frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}=\frac{1}{1+2(\zeta/\omega_n)s+(s/\omega_n)^2}$$

$$polos \quad s=-\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2} \quad , \quad \zeta\le 1$$

$$P(j\omega)=\frac{1}{1-(\omega/\omega_n)^2+j2\zeta(\omega/\omega_n)}$$

$$|P(j\omega)|_{db}=-20\log\left[\left(1-(\omega/\omega_n)^2\right)^2+4\zeta^2(\omega/\omega_n)^2\right]^{1/2}$$

$$\arg\{P(j\omega)\}=-\arctan\left(\frac{2\zeta(\omega/\omega_n)}{1-(\omega/\omega_n)^2}\right)$$

Pares de **polos o ceros complejos conjugados** **6.7 h2**

$$|P(j\omega)|_{db}=-20\log\left[\left(1-(\omega/\omega_n)^2\right)^2+4\zeta^2(\omega/\omega_n)^2\right]^{1/2}$$

Asintotas bajas frecuencias, $\omega << \omega_n$

$$|P(j\omega)|_{db}=-20\log 1=0\ db$$

Asintotas altas frecuencias, $\omega >> \omega_n$

$$|P(j\omega)|_{db}=-20\log(\omega/\omega_n)^2=-40\log(\omega/\omega_n)\ db$$

Corte de asintotas $\Rightarrow \quad \omega=\omega_n \Rightarrow \quad |P(j\omega)|_{db}=-40\log 1=0\ db$

$$\mathrm{Re}\,al:\omega=\omega_n \Rightarrow |P(j\omega_n)|_{db}=-20\log(2\zeta)=\begin{cases}14db & ;\ \zeta=0.1\\ 0db & ;\ \zeta=0.5\\ -3db & ;\ \zeta=0.707\end{cases}$$

Pares de **polos o ceros complejos conjugados**

6.7 h3

$$\arg\{P(j\omega)\} = -\arctan\left(\frac{2\zeta(\omega/\omega_n)}{1-(\omega/\omega_n)^2}\right)$$

Asintotas bajas frecuencias, $\omega << \omega_b$

$$\arg\{P(j\omega)\} = -\arctan(0) = 0^o$$

Asintotas altas frecuencias, $\omega >> \omega_b$

$$\arg\{P(j\omega)\} = -\arctan(0^-) = -180^o$$

$$\mathrm{Re}\,al : \omega = \omega_n \quad \Rightarrow \quad \arg\{P(j\omega_n)\} = -90^o$$

Figure 6.32 (p. 537)

Asymptotic approximation to $20\log_{10}|Q(j\omega)|$, where

$$Q(s) = \frac{1}{1 + (2\zeta/\omega_n)s + s^2/\omega^2{}_n}$$

$\mathrm{v} \Rightarrow \omega$

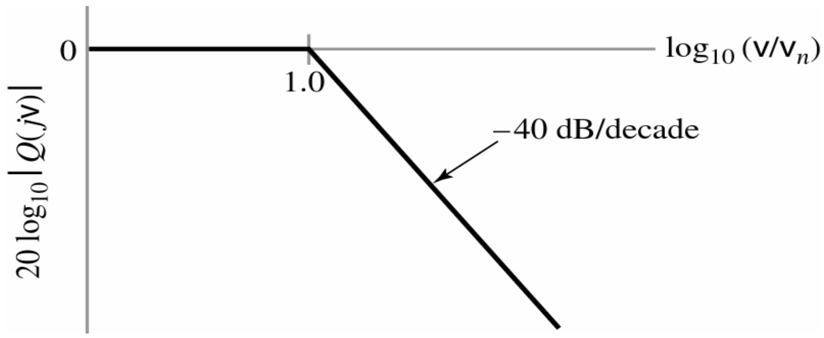
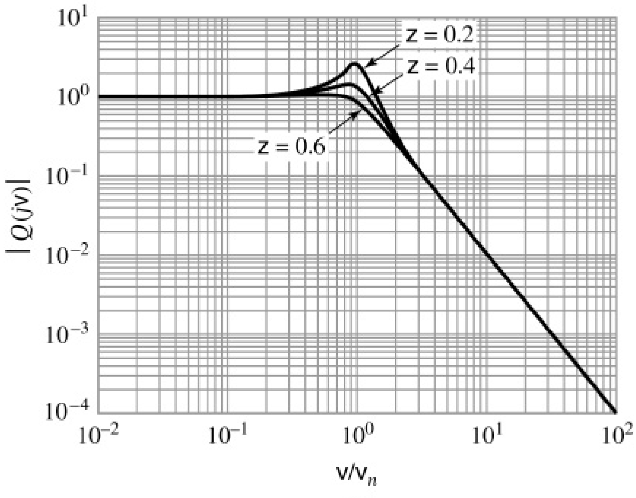


Figure 6.33 (p. 538)
Bode diagram of second-order pole factor for varying ζ
(a) Gain response.
(b) Phase response. $Q(s)=\dfrac{1}{1+(2\zeta/\omega_n)s+s^2/\omega_n^2}$



(a)

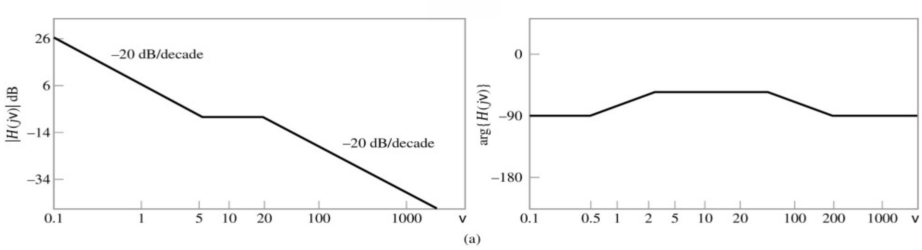
$z \Rightarrow \zeta$

$v \Rightarrow \omega$

Problema 6.23 Dibujar los diagramas de bode para el sistema con función de transferencia :

6.7 i1

$$(a)\quad H(s)=\frac{8s+40}{s(s+20)}$$



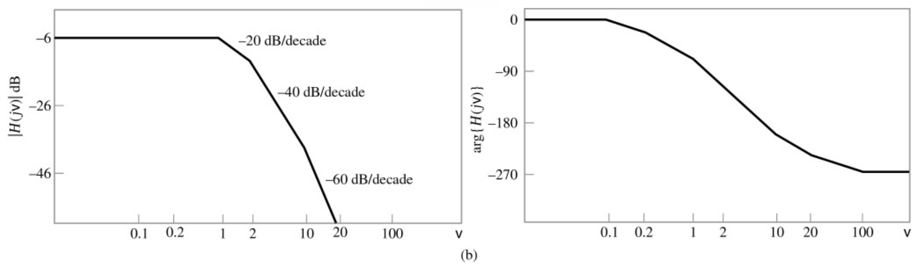
(a)

$v \Rightarrow \omega$

Problema 6.23 Dibujar los diagramas de bode para el sistema con función de transferencia :

6.7 i2

$$(b) \quad H(s) = \frac{10}{(`s+1)(s+2)(s+10)}$$

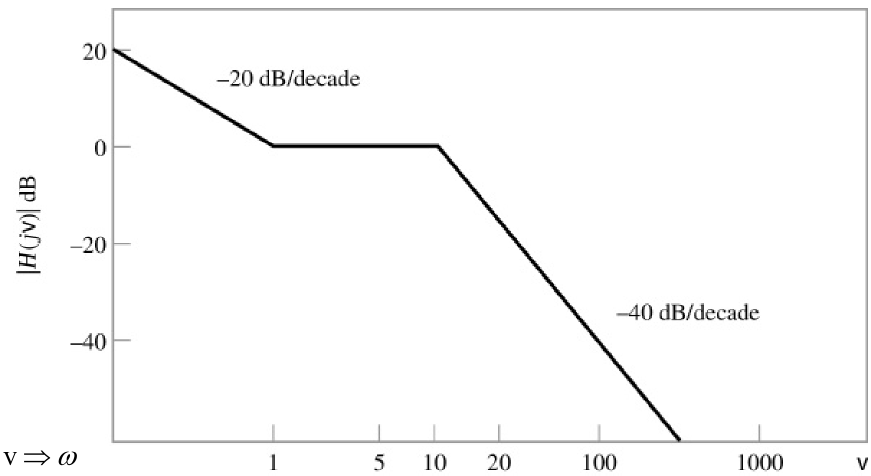


$v \Rightarrow \omega$

Problema 6.23 Dibujar los diagramas de bode (módulo) para el sistema con función de transferencia :

6.7 j

$$H(s) = \frac{100(s+1)}{s(s^2+20s+100)}$$



$v \Rightarrow \omega$

Transformadas de Laplace básicas (1)

Señal	Transformada	ROC
$x(t)=\frac{1}{2\pi j}\int_{\sigma-j\infty}^{\sigma+j\infty}X(s)e^{st}ds$	$X(s)=\int_{-\infty}^{\infty}x(t)e^{-st}dt$	
u(t)	$\frac{1}{s}$	Re{s}>0
t u(t)	$\frac{1}{s^2}$	Re{s}>0
$\delta(t-\tau),\ \tau\geq 0$	e^{-st}	$\forall\ s$
$e^{-at}u(t)$	$\frac{1}{s+a}$	Re{s}>-a
$te^{-at}u(t)$	$\frac{1}{(s+a)^2}$	Re{s}>-a

Transformadas de Laplace básicas (2)

Señal	Transformada	ROC
$x(t)=\frac{1}{2\pi j}\int_{\sigma-j\infty}^{\sigma+j\infty}X(s)e^{st}ds$	$X(s)=\int_{-\infty}^{\infty}x(t)e^{-st}dt$	
$[\cos(\omega_1 t)]u(t)$	$\frac{s}{s^2+\omega_1^2}$	Re{s}>0
$[sen(\omega_1 t)]u(t)$	$\frac{\omega_1}{s^2+\omega_1^2}$	Re{s}>0
$[e^{-at}\cos(\omega_1 t)]u(t)$	$\frac{s+a}{(s+a)^2+\omega_1^2}$	Re{s}>-a
$[e^{-at}sen(\omega_1 t)]u(t)$	$\frac{\omega_1}{(s+a)^2+\omega_1^2}$	Re{s}>-a
$t^n u(t)$	$\frac{n!}{s^{n+1}}$	Re{s}>0

Transformadas de Laplace bilaterales,
para señales distintas de cero para $t \le 0$

Señal	Transformada	ROC
-u(-t)	$\frac{1}{s}$	Re{s}<0
-t u(-t)	$\frac{1}{s^2}$	Re{s}<0
$\delta(t-\tau),\ \tau \le 0$	e^{-st}	$\forall\ s$
$-e^{-at}u(-t)$	$\frac{1}{s+a}$	Re{s}<-a
$-te^{-at}u(-t)$	$\frac{1}{(s+a)^2}$	Re{s}<-a

Propiedades de la transformada de Laplace (1)

Señal	Transformada unilateral	Transformada bilateral	ROC
x(t)	X(s)	X(S)	$s \in R_x$
y(t)	Y(s)	Y(S)	$s \in R_y$
ax(t)+by(t)	aX(s)+bY(s)	aX(s)+bY(s)	al menos $R_x \cap R_y$
x(t-τ)	$e^{-s\tau}X(s)$ $si\ x(t-\tau)u(t) = x(t-\tau)u(t-\tau)$	$e^{-s\tau}X(s)$	R_x
$e^{-s_0 t}x(t)$	$X(s+s_0)$	$X(s+s_0)$	$R_x + \mathrm{Re}\{s_0\}$
x(at)	$\frac{1}{a}X\left(\frac{s}{a}\right),\ a>0$	$\frac{1}{\lvert a\rvert}X\left(\frac{s}{a}\right)$	$\frac{R_x}{\lvert a\rvert}$

Propiedades de la transformada de Laplace (2)

Señal	Transformada unilateral	Transformada bilateral	ROC
x(t)	X(s)	X(S)	$s \in R_x$
y(t)	Y(s)	Y(S)	$s \in R_y$
x(t)*y(t)	X(s)Y(s)	X(s)Y(s)	al menos $R_x \cap R_y$
-t x(t)	$\frac{d}{ds}X(s)$	$\frac{d}{ds}X(s)$	R_x
$\frac{d}{dt}x(t)$	$sX(s)-x(0^+)$	$sX(s)$	al menos R_x
$\int_{-\infty}^{t} x(\tau)d\tau$	$\frac{1}{s}\int_{-\infty}^{0^-} x(\tau)d\tau+\frac{X(s)}{s}$	$\frac{X(s)}{s}$	al menos $R_x \cap \{\text{Re}\{s\}>0\}$

Propiedades de la transformada de Laplace (3)

Teorema del valor inicial $\lim_{s\to\infty} sX(s) = x(0^+)$

Teorema del valor final $\lim_{s\to 0} sX(s) = x(\infty)$ solo si Re(polos)<0

Propiedad de diferenciación unilateral-Forma general

$$\frac{d^n}{dt^n}x(t) \xleftrightarrow{L_u} s^n X(s) - \frac{d^{n-1}}{dt^{n-1}}x(t)\big|_{t=0^+} - \ldots - s^{n-2}\frac{d}{dt}x(t)\big|_{t=0^+} - s^{n-1}x(0^+)$$

Problema 6.26

6.26. A signal $x(t)$ has Laplace transform $X(s)$ as given below. Plot the poles and zeros in the s-plane and determine the Fourier transform of $x(t)$ without inverting $X(s)$.

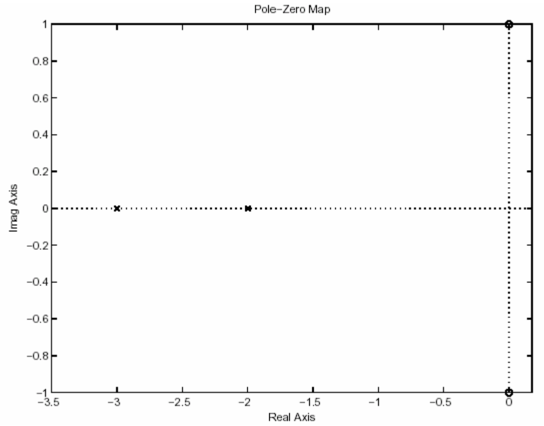
(a) $X(s) = \frac{s^2+1}{s^2+5s+6}$

$$X(s) = \frac{(s+j)(s-j)}{(s+3)(s+2)}$$

zeros at: $\pm j$

poles at: $-3, -2$

$$X(j\omega) = X(s)|_{s=j\omega} = \frac{-\omega^2+1}{-\omega^2+5j\omega+6}$$



Problema 6.27

6.27. Determine the bilateral Laplace transform and ROC for the following signals:

(a) $x(t) = e^{-t}u(t+2)$

$$\begin{aligned} X(s) &= \int_{-\infty}^{\infty} x(t)e^{-st}\,dt \\ &= \int_{-\infty}^{\infty} e^{-t}u(t+2)e^{-st}\,dt \\ &= \int_{-2}^{\infty} e^{-t(1+s)}\,dt \\ &= \frac{e^{2(1+s)}}{1+s} \end{aligned}$$

ROC: Re(s) > -1

Problema 6.28

6.28. Determine the unilateral Laplace transform of the following signals using the defining equation:

(e) $x(t) = \sin(\omega_o t)$

$$\begin{aligned} X(s) &= \int_{0-}^{\infty} \frac{1}{2j}\left(e^{j\omega_o t} - e^{-j\omega_o t}\right) e^{-st}\, dt \\ &= \frac{1}{2j}\left[\int_{0-}^{\infty} e^{t(j\omega_o - s)}\, dt - \int_{0-}^{\infty} e^{-t(j\omega_o + s)}\, dt\right] \\ &= \frac{1}{2j}\left[\frac{-1}{j\omega_o - s} - \frac{1}{j\omega_o + s}\right] \\ &= \frac{\omega_o}{s^2 + \omega_o^2} \end{aligned}$$

Problema 6.30

6.30. Use the basic Laplace transforms and the Laplace transform properties given in Tables D.1 and D.2 to determine the time signals corresponding to the following unilateral Laplace transforms:

(b) $X(s) = e^{-2s} \frac{d}{ds}\left(\frac{1}{(s+1)^2}\right)$

$$\begin{aligned} A(s) = \frac{1}{(s+1)^2} &\quad \stackrel{\mathcal{L}_u}{\longleftrightarrow} \quad a(t) = te^{-t}u(t) \\ B(s) = \frac{d}{ds}A(s) &\quad \stackrel{\mathcal{L}_u}{\longleftrightarrow} \quad b(t) = -ta(t) = -t^2e^{-t}u(t) \\ X(s) = e^{-2s}B(s) &\quad \stackrel{\mathcal{L}_u}{\longleftrightarrow} \quad x(t) = b(t-2) = -(t-2)^2e^{-(t-2)}u(t-2) \end{aligned}$$

Problema 6.32

6.32. Given the transform pair $x(t) \stackrel{\mathcal{L}_u}{\longleftrightarrow} \frac{2s}{s^2+2}$, where $x(t) = 0$ for $t < 0$, determine the Laplace transform of the following time signals:

(a) $x(3t)$

$$x(3t) \stackrel{\mathcal{L}_u}{\longleftrightarrow} \frac{1}{3}X(\frac{s}{3})$$

$$X(s) = \frac{2\frac{s}{3}}{(\frac{s}{3})^2+2} = \frac{6s}{s^2+18}$$

(d) $e^{-t}x(t)$

$$e^{-t}x(t) \stackrel{\mathcal{L}_u}{\longleftrightarrow} X(s+1) = \frac{2(s+1)}{(s+1)^2+2}$$

Problema 6.33

6.33. Use the s-domain shift property and the transform pair $e^{-at}u(t) \stackrel{\mathcal{L}_u}{\longleftrightarrow} \frac{1}{s+a}$ to derive the unilateral Laplace transform of $x(t) = e^{-at}\cos(\omega_1 t)u(t)$.

$$e^{-at}u(t) \stackrel{\mathcal{L}_u}{\longleftrightarrow} \frac{1}{s+a}$$

$$x(t) = e^{-at}\cos(\omega_1 t)u(t) = \frac{1}{2}e^{-at}\left(e^{j\omega_1 t} + e^{-j\omega_1 t}\right)u(t)$$

Using the s-domain shift property:

$$X(s) = \frac{1}{2}\left(\frac{1}{(s-j\omega_1)+a} + \frac{1}{(s+j\omega_1)+a}\right) = \frac{1}{2}\frac{2(s+a)}{(s+a)^2+\omega_1^2} = \frac{(s+a)}{(s+a)^2+\omega_1^2}$$

Problemas 6.35 y 6.36

6.35. Determine the initial value $x(0^+)$ given the following Laplace transforms $X(s)$:

(c) $X(s) = e^{-2s}\frac{6s^2+s}{s^2+2s-2}$

$$x(0^+) = \lim_{s\to\infty} sX(s) = e^{-2s}\frac{6s^3+s^2}{s^2+2s-2} = 0$$

6.36. Determine the final value $x(\infty)$ given the following Laplace transforms $X(s)$:

(b) $X(s) = \frac{s+2}{s^3+2s^2+s}$

$$x(\infty) = \lim_{s\to 0} sX(s) = \frac{s+2}{s^2+2s+1} = 2$$

Problema 6.37

6.37. Use the method of partial fractions to find the time signals corresponding to the following unilateral Laplace transforms:

(b) $X(s) = \frac{2s^2+10s+11}{s^2+5s+6}$

$$X(s) = \frac{2s^2+10s+11}{s^2+5s+6} = 2 - \frac{1}{(s+2)(s+3)}$$

$$\frac{1}{(s+2)(s+3)} = \frac{A}{s+2} + \frac{B}{s+3}$$

$$0 = A + B$$

$$1 = 3A + 2B$$

$$X(s) = 2 - \frac{1}{s+2} + \frac{1}{s+3}$$

$$x(t) = 2\delta(t) + \left[e^{-3t} - e^{-2t}\right]u(t)$$

Problema 6.38

6.38. Determine the forced and natural responses for the LTI systems described by the following differential equations with the specified input and initial conditions:

(b) $\frac{d^2}{dt^2}y(t) + 5\frac{d}{dt}y(t) + 6y(t) = -4x(t) - 3\frac{d}{dt}x(t),\ \ y(0^-) = -1,\ \frac{d}{dt}y(t)\big|_{t=0^-} = 5,\ \ x(t) = e^{-t}u(t)$

$$Y(s)(s^2 + 5s + 6) - 5 + s + 5 = (-4 - 3s)\frac{1}{s+1}$$

$$\begin{aligned}
Y(s) &= \frac{-1}{(s+1)(s+2)(s+3)} + \frac{s}{(s+2)(s+3)} \\
&= Y^f(s) + Y^n(s) \\
Y^f(s) &= \frac{-0.5}{s+1} + \frac{-2}{s+2} + \frac{2.5}{s+3} \\
y^f(t) &= \left(-0.5e^{-t} - 2e^{-2t} + 2.5e^{-3t}\right)u(t) \\
Y^n(s) &= \frac{-2}{s+2} + \frac{3}{s+3} \\
y^n(t) &= \left(-2e^{-2t} + 3e^{-3t}\right)u(t)
\end{aligned}$$

Problema 6.41

6.41. Determine the bilateral Laplace transform and the corresponding ROC for the following signals:

(b) $x(t) = e^t\cos(2t)u(-t) + e^{-t}u(t) + e^{t/2}u(t)$

$$X(s) = -\frac{s-1}{(s-1)^2+4} + \frac{1}{s+1} + \frac{1}{s-\frac{1}{2}} \qquad \text{ROC: } 0.5 < \text{Re}(s) < 1$$

(c) $x(t) = e^{3t+6}u(t+3)$

$$\begin{aligned}
x(t) &= e^{-3}e^{3(t+3)}u(t+3) \\
a(t) = e^{3t}u(t) &\overset{\mathcal{L}}{\longleftrightarrow} A(s) = \frac{1}{s-3} \\
b(t) = a(t+3) &\overset{\mathcal{L}}{\longleftrightarrow} B(s) = e^{3s}A(s) = e^{3s}\frac{1}{s-3} \\
X(s) &= \frac{e^{3(s-1)}}{s-3}
\end{aligned}$$

ROC: $\text{Re}(s) > 3$

Problema 6.43

6.43. Use the method of partial fractions to determine the time signals corresponding to the following bilateral Laplace transforms:

(b) $X(s) = \frac{4s^2+8s+10}{(s+2)(s^2+2s+5)}$

$$X(s) = \frac{2}{s+2} + \frac{2(s+1)}{(s+1)^2+2^2} + \frac{-2}{(s+1)^2+2^2}$$

(i) with ROC $\text{Re}(s) < -2$ (left-sided)

$$x(t) = \left(-2e^{-2t} - 2e^{-t}\cos(2t) + e^{-t}\sin(2t)\right)u(-t)$$

(ii) with ROC $\text{Re}(s) > -1$ (right-sided)

$$x(t) = \left(2e^{-2t} + 2e^{-t}\cos(2t) - e^{-t}\sin(2t)\right)u(t)$$

(iii) with ROC $-2 < \text{Re}(s) < -1$ (two sided)

$$x(t) = 2e^{-2t}u(t) + \left(-2e^{-t}\cos(2t) + e^{-t}\sin(2t)\right)u(-t)$$

Problema 6.45

6.45. A system has transfer function $H(s)$ as given below. Determine the impulse response assuming (i) that the system is causal, and (ii) that the system is stable.

(a) $H(s) = \frac{2s^2+2s-2}{s^2-1}$

$$H(s) = 2 + \frac{1}{s+1} + \frac{1}{s-1}$$

(i) system is causal

$$h(t) = 2\delta(t) + \left(e^{-t} + e^{t}\right)u(t)$$

(ii) system is stable

$$h(t) = 2\delta(t) + e^{-t}u(t) + -e^{t}u(-t)$$

Problema 6.47

6.47. The relationship between the input $x(t)$ and output $y(t)$ of a causal system is described by the differential equation given below. Use Laplace transforms to determine the transfer function and impulse response of the system.

(b) $\frac{d^2}{dt^2}y(t) + 5\frac{d}{dt}y(t) + 6y(t) = x(t) + \frac{d}{dt}x(t)$

$$\begin{aligned}
Y(s)(s^2+5s+6) &= X(s)(1+s) \\
H(s) &= \frac{s+1}{(s+3)(s+2)} \\
&= \frac{-1}{s+2} + \frac{2}{s+3} \\
h(t) &= \left(2e^{-3t} - e^{-2t}\right)u(t)
\end{aligned}$$

Problema 6.48

6.48. Determine a differential equation description for a system with the following transfer function.

(c) $H(s) = \frac{2(s-2)}{(s+1)^2(s+3)}$

$$\begin{aligned}
Y(s)(s^3+5s^2+7s+3) &= X(s)(3s^2+6s+3) \\
\frac{d^3}{dt^3}y(t) + 5\frac{d^2}{dt^2}y(t) + 7\frac{d}{dt}y(t) + 3y(t) &= 2\frac{d}{dt}x(t) - 4x(t)
\end{aligned}$$

Problema 6.49

(b) Determine the transfer function, impulse response, and differential equation descriptions for a stable LTI system represented by the following state variable descriptions:

(i) $\mathbf{A} = \begin{bmatrix} -1 & 1 \\ 0 & -2 \end{bmatrix}$, $\mathbf{b} = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$, $\mathbf{c} = \begin{bmatrix} 1 & 2 \end{bmatrix}$, $D = [0]$

$$\begin{aligned} H(s) &= \mathbf{c}(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{b} + D \\ &= \frac{s+3}{s^2+3s+1} \\ &= \frac{2}{s+1} + \frac{-1}{s+2} \end{aligned}$$

$$\begin{aligned} h(t) &= \left(2e^{-t} - e^{-2t}\right) u(t) \\ \frac{d^2}{dt^2}y(t) + 3\frac{d}{dt}y(t) + y(t) &= \frac{d}{dt}x(t) + 3x(t) \end{aligned}$$

Problema 6.50

6.50. Determine whether the systems described by the following transfer functions are(i) both stable and causal, and (ii) whether a stable and causal inverse system exists:

(b) $H(s) = \frac{s^2+2s-3}{(s+3)(s^2+2s+5)}$

$$H(s) = \frac{s-1}{s^2+2s+5}$$

zero at: 1

poles at: $-1 \pm 2j$

(i) All poles are in the LHP, and with ROC: Re(s) > -1, the system is both stable and causal.
(ii) Not all zeros are in the LHP, so no stable and causal inverse system exists.

(a) $H(s) = \frac{(s+1)(s+2)}{(s+1)(s^2+2s+10)}$

$$H(s) = \frac{s+2}{s^2+2s+10}$$

zero at: -2

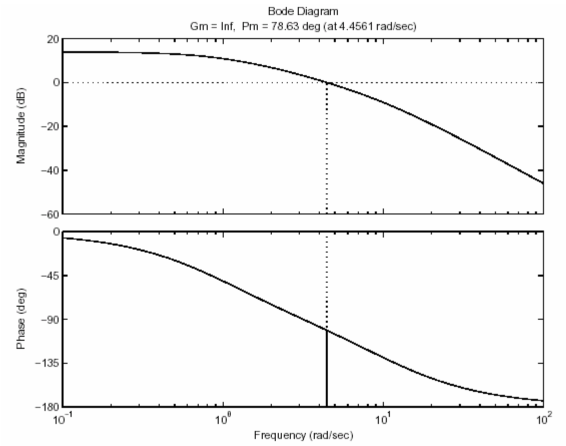
poles at: $-1 \pm 3j$

(i) All poles are in the LHP, and with ROC: Re(s) > -1, the system is both stable and causal.
(ii) All zeros are in the LHP, so a stable and causal inverse system exists.

Problema 6.55 a

6.55. Sketch the Bode diagrams for the systems described by the following transfer functions.

(a) $H(s) = \frac{50}{(s+1)(s+10)}$



Problema 6.55 b

6.55. Sketch the Bode diagrams for the systems described by the following transfer functions.

(b) $H(s) = \frac{20(s+1)}{s^2(s+10)}$